

Fisher information of two superconducting charge qubits under dephasing noisy

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Abstract

The dynamics of a charged two-qubit system prepared initially in a maximum entangled state is discussed, where each qubit interacts independently with a dephasing channel. The Fisher information is used to estimate the channel and the energy parameters. Moreover, the contribution from different parts of Fisher information is discussed. We show that, the degree of estimation of any parameter depends on the initial value of this parameter. It turns out that the upper bounds of Fisher information with respect to the dephasing parameter increase and that corresponding to coupling parameter decreases as the initial energy of each qubit increases.

1 Introduction

Quantum Fisher information is considered as one of the important measurements in the estimation theory [1, 2], which characterizes the sensitivity of a given system with respect to the changes in one of its parameters. Recently, in the context of quantum estimation theory Fisher information has paid some attentions. For example, Z. Qiang et. al [3] examined the problem of parameter estimation for two initially entangled qubits subject to decoherence. Xiao et. al.[4] showed that the partial measurements can greatly enhance the quantum Fisher information teleportation under decoherence. F. Fröwis et. al proposed an experimentally accessible scheme to determine lower bounds on the quantum Fisher information [5]. Quantum Fisher information of the Greenberger-Horne-Zeilinger state in decoherence channels was discussed by Ma et.al [6]. The dynamic of quantum Fisher information of W state in the three basic decoherence channels was studied by Ozaydin [7]. The properties of quantum Fisher information in a general superposition of a 3-qubit GHZ state and two W states were investigated by Yi et. al [9]. The time evolution of the quantum Fisher information of a system whose the dynamics was described by the phase-damped model is discussed by Obada et. al [10]. The effect of the Unruh parameter on the precision of estimating the channel parameter was discussed in [11]. Recently, Metwally[12] employed the Fisher information to estimate the teleported and the gained parameters by using an accelerated channel.

However, investigating the Fisher information for the charged qubits has not paid much attention. Therefore, we are motivated to use the Fisher information to estimate the initial parameters of the charged qubits system; the Josephson energies parameters and the mutual coupling energy between the two qubits and the channel noise parameter.

This paper is organized as follows: In Sec. 2, we present the model and its solution under dephasing noisy. In Sec. 3, we estimate the charged system's parameters and the noisy channel parameter by means of Fisher information, where analytical forms are introduced. We summarize our results and conclusion in Sec. 4.

2 The model and its solution

Charged qubit represents one of the most promising particles in quantum computation and information. There are several studies have been introduced to investigate the properties of entangled charged qubits. For example, Liao et. al, [13] investigated the entanglement between two Josephson charge qubits. The dynamics of charge qubits coupled to a nonmechanical resonator under the influence a phonon bath is discussed by Abdel-Aty et.al [14]. Dynamics of the quantum deficit of two charged qubit is investigated in [15]. The possibility of using charged qubit to perform quantum teleportation is investigated by Metwally [16].

However, the Hamiltonian which can be used to describe a two charged qubit system is defined as [17, 18, 19]:

$$\mathcal{H} = -\frac{1}{2} \left\{ \kappa_1 \sigma_z^{(1)} \otimes I_2 + \kappa_2 I_1 \otimes \sigma_z^{(2)} + E_{J_1} \sigma_x^{(1)} \otimes I_2 + E_{J_2} I_1 \otimes \sigma_x^{(2)} - 2E_m \sigma_z^{(1)} \otimes \sigma_z^{(2)} \right\} \quad (1)$$

where $\kappa_1 = 2E_{c_1}(1 - 2n_{g_1}) + E_m(1 - 2n_{g_2})$, $\kappa_2 = 2E_{c_2}(1 - 2n_{g_2}) + E_m(1 - 2n_{g_1})$, I_1, I_2 are the unit operators for the first and the second qubit, respectively and E_{c_i} , E_{J_i} are the charging and Josephson energies for the first and the second qubit, respectively. The mutual coupling energy between the two qubits is defined by the parameter E_m . The operators $\sigma_{z,x}^{(i)}$, $i = 1, 2$ represent the Pauli operators for the two qubits, respectively.

Assume that the charged qubit state is prepared in the maximum Bell state $\rho_{\psi^+} = |\psi^+\rangle\langle\psi^+|$, $|\psi^+\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$. In our investigation, we consider the following assumptions:

- (i) The two charged qubits are identical, namely, $E_{J_1} = E_{J_2} = E_J$.
- (ii) The system is assumed to be is in the degenerate point, namely, $n_{g_1} = n_{g_2} = 0.5$.
- (iii) The noise channels are identical, i.e., having the same strength.

Under these considerations, the Hamiltonian (1) reduces to be,

$$\mathcal{H} = -\frac{1}{2} \left\{ E_{J_1} \sigma_x^{(1)} \otimes I_2 + E_{J_2} I_1 \otimes \sigma_x^{(2)} - 2E_m \sigma_z^{(1)} \otimes \sigma_z^{(2)} \right\}. \quad (2)$$

The master equation which governed the system is given by:

$$\frac{d\rho(t)}{dt} = -i[\mathcal{H}, \rho] + \frac{\Gamma}{8} \sum_{j=1,2} (2\sigma_z^{(j)} \rho \sigma_z^{(j)} - \sigma_z^{(j)} \sigma_z^{(j)} \rho - \rho \sigma_z^{(j)} \sigma_z^{(j)}) \quad (3)$$

where, Γ is the dephasing parameter. By solving this master equation, one obtains $\rho(t)$, which describes the evolution of the charged system. This state is described by a square matrix of order 4 with elements are given by:

$$\begin{aligned} \varrho_{11} &= \varrho_{44} = \Upsilon_1 \left\{ 2\lambda_{123} \exp[2\Gamma t] - \lambda_2 \mu_- R_1^+ - \lambda_1 \mu_+ R_2^+ \right\}, \\ \varrho_{22} &= \varrho_{33} = \Upsilon_1 \left\{ 2\lambda_{123} \exp[2\Gamma t] + \lambda_2 \mu_- R_1^+ + \lambda_1 \mu_+ R_2^+ \right\}, \\ \varrho_{14} &= \varrho_{41} = \Upsilon_1 \left\{ 2\lambda_{123} \exp[-2\Gamma t] + \lambda_2 \mu_- R_1^- + \lambda_1 \mu_+ R_2^- \right\}, \\ \varrho_{23} &= \varrho_{32} = \Upsilon_1 \left\{ 2\lambda_{123} \exp[-2\Gamma t] - \lambda_2 \mu_- R_1^- - \lambda_1 \mu_+ R_2^- \right\}, \\ \varrho_{12} &= \varrho_{13} = \varrho_{42} = \varrho_{43} = \Upsilon_2 \left\{ 2E_m \left[\cosh(\sqrt{2}\lambda_2 t) - \cos(\sqrt{2}\lambda_1 t) \right] \right. \\ &\quad \left. + i\sqrt{2} \left[\lambda_2 \sinh(\sqrt{2}\lambda_2 t) + \lambda_1 \sin(\sqrt{2}\lambda_1 t) \right] \right\}, \\ \varrho_{21} &= \varrho_{31} = \varrho_{24} = \varrho_{34} = \varrho_{12}^* \end{aligned} \quad (4)$$

where,

$$\begin{aligned} \Upsilon_1 &= \exp(-2\Gamma t)/8\lambda_{123}, \quad \Upsilon_2 = \frac{E_j \exp(-2\Gamma t)}{8\lambda_3}, \quad \lambda_{123} = \lambda_1 \lambda_2 \lambda_3, \\ \mu_{\pm} &= \lambda_3 \pm (\Gamma^2 + E_m^2 - E_j^2) \\ R_1^{\pm} &= \sqrt{2}\Gamma \sin(\sqrt{2}\lambda_1 t) \pm \lambda_2 \cos(\sqrt{2}\lambda_1 t), \\ R_2^{\pm} &= \sqrt{2}\Gamma \sinh(\sqrt{2}\lambda_2 t) \pm \lambda_2 \cosh(\sqrt{2}\lambda_2 t), \\ \lambda_{1,2} &= \sqrt{\lambda_3 \pm (E_m^2 + E_j^2 - \Gamma^2)}, \\ \lambda_3 &= \sqrt{E_m^4 + (E_j^2 - \Gamma^2)^2 + 2E_m(E_j^2 + \Gamma^2)^2}. \end{aligned} \quad (5)$$

The eigenvalues of the density operator (4) are given by:

$$\epsilon_1 = \varrho_{11} - \varrho_{14}, \quad \epsilon_2 = \varrho_{22} - \varrho_{23}, \quad \epsilon_{3,4} = -2\sqrt{\alpha^2 + \beta^2} \mu_{\pm} \quad (6)$$

and the corresponding eigenvectors are given by:

$$\begin{aligned} |V_1\rangle &= \frac{1}{\sqrt{2}} (-1, 0, 0, 1), \quad |V_2\rangle = \frac{1}{\sqrt{2}} (0, -1, 1, 0), \\ |V_3\rangle &= \frac{1}{\sqrt{2(1 + \mu_-^2)}} (1, \mu_- \exp(-i\theta), \mu_- \exp(-i\theta), 1), \\ |V_4\rangle &= \frac{1}{\sqrt{2(1 + \mu_+^2)}} (1, \mu_+ \exp(-i\theta), \mu_+ \exp(-i\theta), 1), \end{aligned} \quad (7)$$

where,

$$\begin{aligned}
\theta &= \tan^{-1} \left(\frac{\beta}{\alpha} \right), \\
\alpha &= \frac{E_j E_m \exp(-2\Gamma t)}{4\lambda_3} \left[\cosh \left(\sqrt{2}\lambda_2 t \right) - \cos \left(\sqrt{2}\lambda_1 t \right) \right], \\
\beta &= \frac{\sqrt{2}E_j \exp(-2\Gamma t)}{8\lambda_3} \left[\lambda_2 \sinh \left(\sqrt{2}\lambda_2 t \right) + \lambda_1 \sin \left(\sqrt{2}\lambda_1 t \right) \right] \\
\mu_{\pm} &= \frac{1}{4\sqrt{\alpha^2 + \beta^2}} \left(-\varrho_{11} - \varrho_{14} + \varrho_{22} + \varrho_{23} \pm \sqrt{(\varrho_{11} + \varrho_{14} + \varrho_{22} + \varrho_{23})^2 + 16\varrho_{12}\varrho_{21}} \right).
\end{aligned} \tag{8}$$

3 Fisher information

In the estimation theory, the quantum Fisher information plays a central role. There are some parameters cannot be quantified directly, so quantum Fisher information can be used to estimate these parameters [20]. Let η represents the parameter to be estimated and the density operator of the spectral decomposition is given as $\varrho_{\eta} = \sum_{i=1}^n \mathcal{E}_i |\mathcal{V}_i\rangle \langle \mathcal{V}_i|$ where, \mathcal{E}_i and $|\mathcal{V}_i\rangle$ are the eigenvalues and eigenvectors of the density operator ρ_{η} , respectively. The Fisher information corresponding to the parameter η is given by (see [1, 11, 21, 22]):

$$\mathcal{F}_{\eta} = \mathcal{F}_C^{\eta} + \mathcal{F}_P^{\eta} - \mathcal{F}_M^{\eta}, \tag{9}$$

where,

$$\begin{aligned}
\mathcal{F}_C^{\eta} &= \sum_{i=1}^n \frac{1}{\mathcal{E}_i} \left(\frac{\partial \mathcal{E}_i}{\partial \eta} \right)^2, & \mathcal{E}_i \neq 0 \\
\mathcal{F}_P^{\eta} &= 4 \sum_{i=1}^n \mathcal{E}_i \left(\left\langle \frac{\partial \mathcal{V}_i}{\partial \eta} \middle| \frac{\partial \mathcal{V}_i}{\partial \eta} \right\rangle - \left\langle \mathcal{V}_i \middle| \frac{\partial \mathcal{V}_i}{\partial \eta} \right\rangle \right), \\
\mathcal{F}_M^{\eta} &= 8 \sum_{i \neq j}^n \frac{\mathcal{E}_i \mathcal{E}_j}{\mathcal{E}_i + \mathcal{E}_j} \left| \left\langle \mathcal{V}_i \middle| \frac{\partial \mathcal{V}_j}{\partial \eta} \right\rangle \right|^2, & \mathcal{E}_i + \mathcal{E}_j \neq 0
\end{aligned} \tag{10}$$

Using the eigenvalues ϵ_i and the eigenvectors $|V_i\rangle$ of the density operator (4), one obtains the explicit form of Fisher information for arbitrary parameter η as:

$$\mathcal{F}_C^{\eta} = \sum_{i=1}^4 \frac{1}{\epsilon_i} \left(\frac{\partial \epsilon_i}{\partial \eta} \right)^2, \tag{11}$$

$$\begin{aligned}
\mathcal{F}_P^{\eta} &= \frac{4\epsilon_3 [\mu'^2_- + \mu_-^2 \theta'^2]}{(1 + \mu_-^2)^2} + \frac{4\epsilon_4 [\mu'^2_+ + \mu_+^2 \theta'^2]}{(1 + \mu_+^2)^2}, \\
\mathcal{F}_M^{\eta} &= \frac{8\epsilon_3 \epsilon_4}{(\epsilon_3 + \epsilon_4) (1 + \mu_+^2)^3 (1 + \mu_-^2)^3} \left\{ (\mu_+ - \mu_-)^2 \left[(1 + \mu_-^2)^2 \mu'^2_+ + (1 + \mu_+^2)^2 \mu'^2_- \right] \right. \\
&\quad \left. + 2\mu_+^2 \mu_-^2 (1 + \mu_+^2)^2 (1 + \mu_-^2)^2 \theta'^2 \right\},
\end{aligned} \tag{12}$$

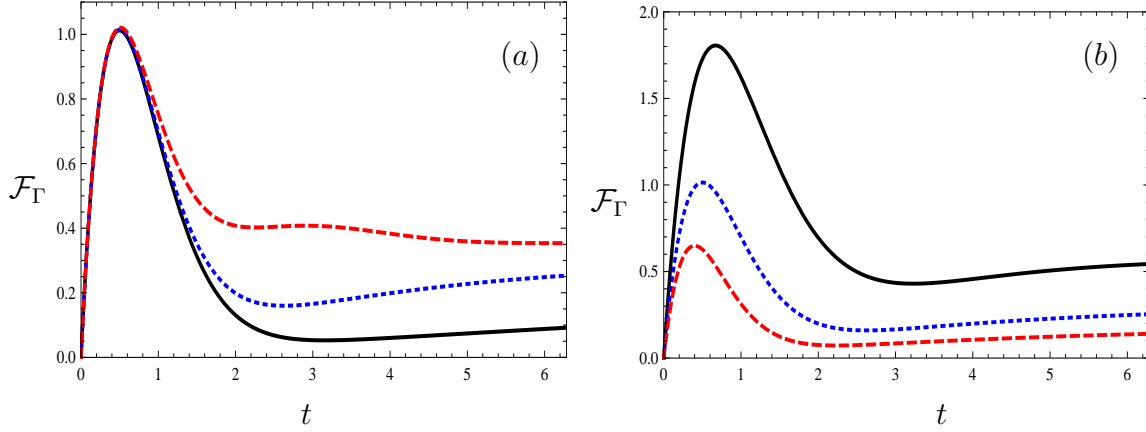


Figure 1: Fisher information \mathcal{F}_Γ as a function of time t . In (a), we set $\Gamma = 0.4$ where, black-solid, blue-dotted, red-dot dashed curves represent $E_j = E_m = 0.05, 0.1, 0.2$, respectively. For (b), we set $E_j = E_m = 0.1$ where, black-solid, blue-dotted, red-dotdashed curves represent $\Gamma = 0.3, 0.4, 0.5$, respectively.

where $\mu'_\pm = \frac{\partial \mu_\pm}{\partial \eta}$ and $\theta' = \frac{\partial \theta}{\partial \eta}$. In the following subsection, we investigate the initial parameters of the charged qubit E_j and E_m and the channel noisy parameter Γ .

3.1 Fisher information with respect to Γ

In this subsection, we estimate the parameter Γ by evaluating the corresponding Fisher information, namely \mathcal{F}_Γ . In Fig.(1a), we investigate the effect of E_i on the dynamics of \mathcal{F}_Γ . It is clear that, at $t = 0$, \mathcal{F}_Γ is zero. However, as soon as the interaction is switched on, the Fisher information increases suddenly to reach its maximum value. For further values of t , \mathcal{F}_Γ decreases fast to reach its minimum value. The minimum values depend on the values of E_i , where these minimum values are larger for larger values of E_i . On the other hand, this behavior is unchanged for further values of t .

In Fig.(1b), we investigate the effect of different initial values of Γ on the \mathcal{F}_Γ , where we set $E_j = E_m = 0.1$. It is evident that, for small values of t , a similar behavior is depicted as that shown in Fig.(1a), i.e., the sudden increasing behavior is displayed as soon as the interaction is switched on. Fisher information reaches its upper bound as t increases. However, the upper bounds depend on the initial values of Γ . It is clear that, as the initial values of Γ increases, the upper bounds of \mathcal{F}_Γ increases. For further values of t , the Fisher information decreases fast to reach its lower bounds at different values of t depending on the initial values Γ . Also, \mathcal{F}_Γ behaves continently as the interaction time increases.

From Fig.(1), one concludes that, for small range of the interaction time, one can estimate the parameter Γ , with high probability. One can increase, the degree of estimation by starting with a larger values of the parameter Γ or E_i parameters. The larger values of interaction time has no effect on the degree of estimation.

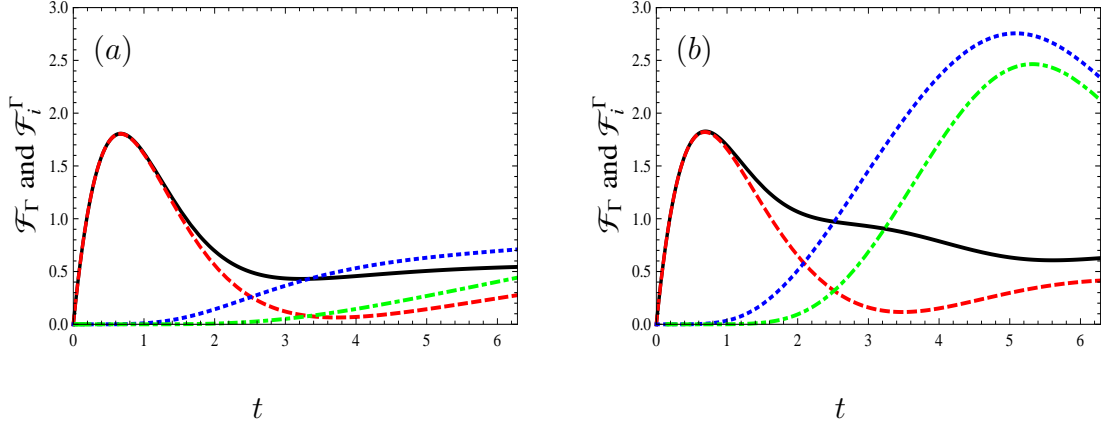


Figure 2: Fisher information \mathcal{F}_Γ and its components \mathcal{F}_i^Γ , $i = C, P, M$ as a function of time t when $\Gamma = 0.4$. The black-solid, red-dotted, blue-dashed and green-dotdashed curves for $\mathcal{F}_\Gamma, \mathcal{F}_C^\Gamma, \mathcal{F}_P^\Gamma$ and \mathcal{F}_M^Γ , respectively. We set in (a) $E_j = E_m = 0.1$ and in (b) $E_j = E_m = 0.2$.

In Fig.(2), we investigate the dynamics of the different parts of the \mathcal{F}_Γ ; the classical Fisher information \mathcal{F}_C , the pure Fisher information \mathcal{F}_P and the mixture of pure state \mathcal{F}_M on the behavior of \mathcal{F}_Γ . It is clear that, the for small values of t the \mathcal{F}_C has the the large contribution. However, as t increases, the pure part \mathcal{F}_P and the mixture part \mathcal{F}_M increase, therefor the Fisher information \mathcal{F}_Γ decreases. The effect of the energy parameters E_j , and the coupling parameter, E_m can be seen by comparing Fig.(2a) and Fig.(2b), where the the rate of increasing \mathcal{F}_P and \mathcal{F}_M in Fig.(2a) is smaller than that depicted in Fig.(2b). This shows that as the energy and the coupling parameters increase, the entangled charged state turns into a product pure states and the possibility of construct the state of the charged qubit as a mixture of pure states increases.

From Fig.(2), we can see that as soon as the interaction is switched on, the classical part \mathcal{F}_C increases, namely, the entangled qubits lose their quantum correlation. For larger time there are some pure states are generated and consequently the pure Fisher information increases. Also, the initial state behaves as a mixture of pure state, namely \mathcal{F}_M increases.

3.2 Fisher information with respect to E_j

Fig.(3a) displays the behavior of Fisher information corresponding to the energy parameter E_i , where we set $E_1 = E_2 = E$. In Fig.(3a), we fix the value of the depshing parameter Γ and different initial energies are considered. It is clear that, \mathcal{F}_{E_j} increases as soon as the interaction is switched on. The upper bounds depend on the initial values of E , where for small initial values of E , the upper bounds of Fisher information \mathcal{F}_{E_j} are larger than those depicted for small values of E .

The effect of different values of the depshing parameter Γ is displayed in Fig.(3b), where different values of Γ are considered. It is clear that, as Γ increases, the Fisher information \mathcal{F}_{E_j} decreases. Moreover, for small values of the depashing parameter, Fisher information increases

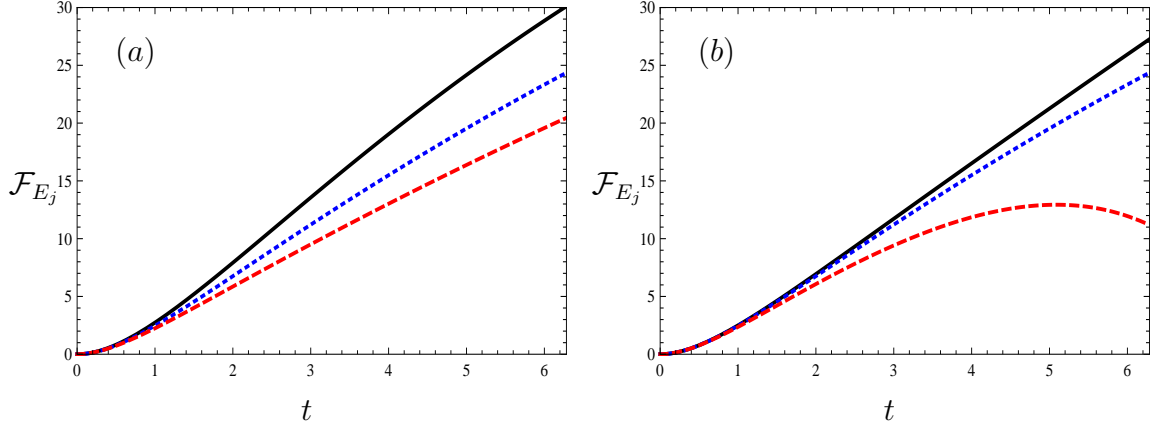


Figure 3: Fisher information \mathcal{F}_{E_j} as a function of time t . The same parameters and properties in Fig. 1 are used

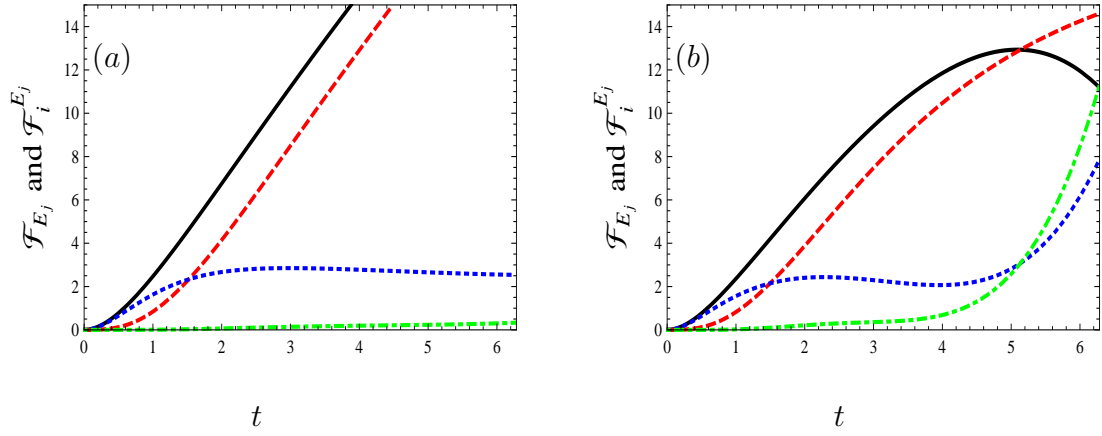


Figure 4: Fisher information \mathcal{F}_{E_j} and its components $\mathcal{F}_i^{E_j}$, $i = C, P, M$ as a function of time t , with the same parameters in Fig. 2

as t increases. However as Γ increases further, the Fisher information decreases.

The effect of the different part of the Fisher information are displayed in Fig.(4a), where the classical part has the larger contribution, while the pure and mixed parts have a small effect. This behavior is changed dramatically as one increases the energy parameter E , where as the interaction time t increases, the pure and the mixed parts of Fisher information rapidly increasing. This explain that why the upper bounds of the Fisher information at $E = 0, 1$ is larger than that displayed at $E = 0.2$. Also, as it is shown in Fig.(4b), the charged qubit turns into product pure state and the possibility of representing it as a mixture of pure states increases as the interaction time increases.

3.3 Estimation of the coupling parameter E_m

To estimate the coupling parameter E_m , we evaluate the corresponding Fisher information \mathcal{F}_{E_m} . The effect of different initial coupling on the Fisher information \mathcal{F}_{E_m} is displayed in Fig.(5a),

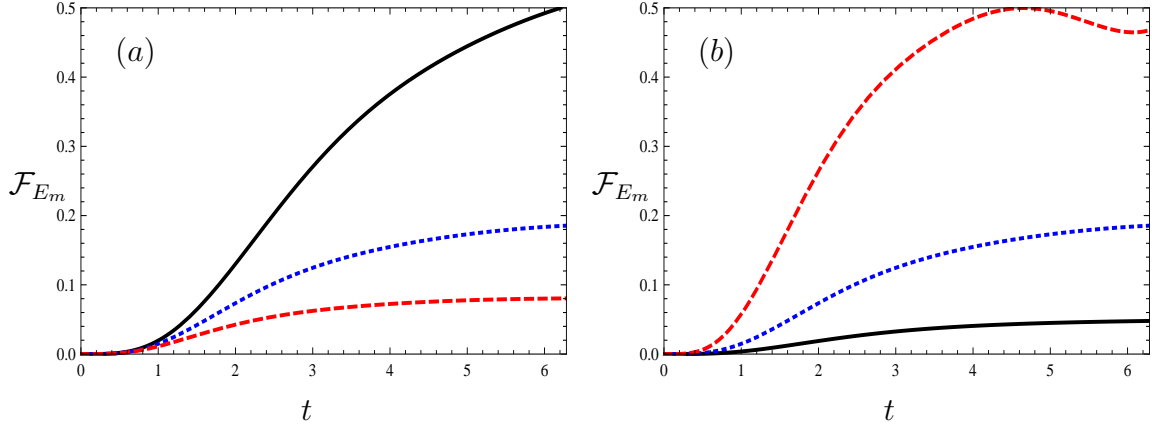


Figure 5: Fisher information \mathcal{F}_{E_m} as a function of time t . The same parameters and properties in Fig. 1 are used

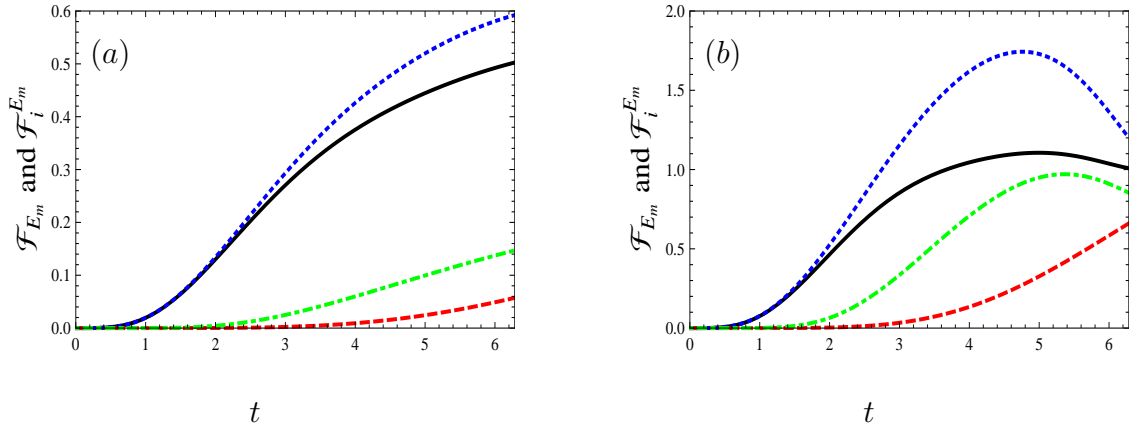


Figure 6: Fisher information \mathcal{F}_{E_m} and its components $\mathcal{F}_i^{E_m}$, $i = C, P, M$ as a function of time t , with the same parameters in Fig. 2

where we fixed the values of the dephasing end energy parameters. It is evident that, as soon as the interaction is switched on, the Fisher information increases \mathcal{F}_{E_m} increases as t increases. However, the upper bounds of \mathcal{F}_{E_m} , depend on the coupling parameter, where its upper bounds are small for larger values the coupling parameter. For larger values of the energy parameter ($E = 0.2$), the behavior is completely different. It is clear that, the Fisher information increases as the initial values of the coupling parameter increases (see Fig.(5b)). This shows that, to increase Fisher information of the coupling parameter, one has to increase the energy of the charged qubit and starting with a larger value of the coupling parameter

The effect of the different parts of the Fisher information is shown in Fig.(6a) and Fig.(6b). It is evident that, the pure part \mathcal{F}_P has the largest contribution, while the classical part \mathcal{F}_C has the smallest contribution. However as one increases the energy and coupling parameters, the classical part and the mixed part increases also for larger interaction time, the pure part decreases. This explain the decreases of the upper bounds of the Fisher information in Fig.(6b).

From Fig.(6) one can conclude that one can increase the possibility of estimating the coupling parameter either by starting with small values of this parameter with small energy, or by larger initial value of the coupling and energy parameters. Also, the possibility of behaving the singlet state as pure increases for smaller values of the energy and the coupling parameters.

4 conclusion

In this contribution, we investigated the dynamics of a charged two-qubit system initially prepared in a maximum entangled state. Each particle interacts independently with a noisy dephasing channel. The final state of the charged qubits was obtained analytically. We estimated the channel parameter as well as the energy parameters by means of Fisher information.

We showed that, Fisher information with respect to the energy increases suddenly as soon as the interaction is switched on to reach its upper bounds. The upper bounds of Fisher information depend on the interaction time, the initial values of the qubits' energy each the charged qubit. For further values of the interaction time, the Fisher information decreases gradually, where the decay rate increases as the energy increases. The long-lived behavior Fisher information with respect to the noisy parameter is clearly depicted for larger values of interaction time. Meanwhile, Fisher information with respect to the energies of the charged qubits increases suddenly as the interaction time increases with larger values of the noisy strength. On the other hand the behavior of Fisher information with respect to the mutual coupling constant increases gradually as the noisy strength increases.

The effect of the different parts of the Fisher information is discussed for all cases. We showed that classical part has the larger contribution on the total Fisher information with respect to the noisy and energy parameters, while for the mutual coupling the pure part has the largest effect. This result is changed for larger values of the initial energy, where the classical has a largest effect on the mutual parameter's information.

From the behavior of the different parts of Fisher information, one deduced when the final state turns into a separable pure state. It is clear that, the initial charged qubit system is robust against the larger energy, since there still a contribution from the pure and mixed parts.

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